DESCRIBING FUNC. ANALYSIS OF NLTI SYSTEMS

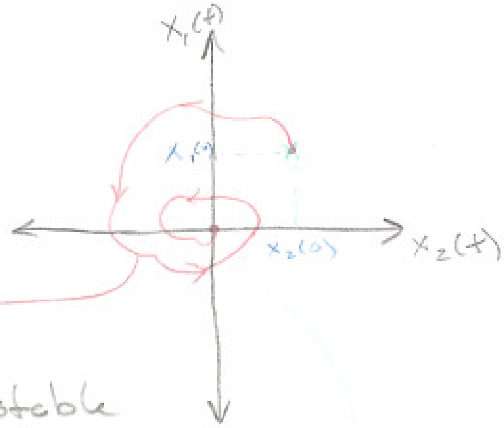
note: No "t" in equation

Phase plane Analysis

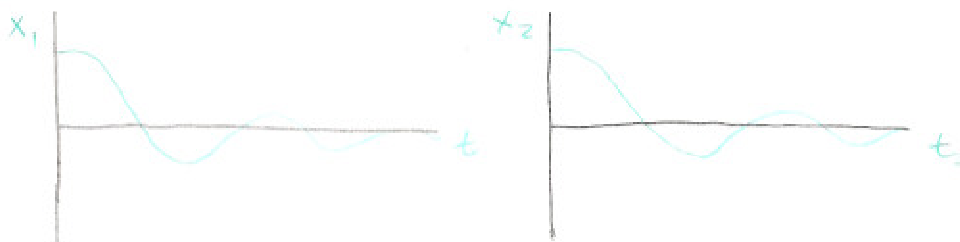
$$\dot{x}_1 = f(x_1, x_2, t) \qquad x_1(0)$$
$$\dot{x}_2 = f(x_1, x_2, t) \qquad x_2(0)$$

$x_1 \in \mathbb{R}$

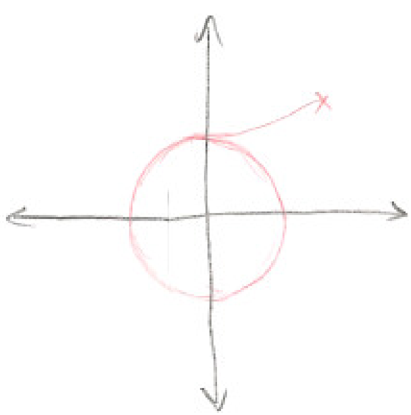
$x_2 \in \mathbb{R}$



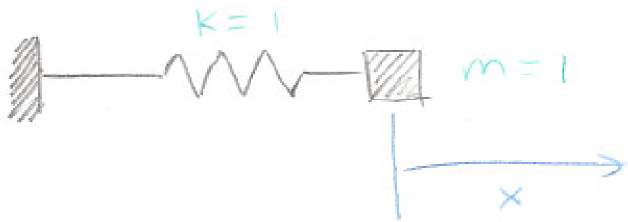
If the system is stable then we will see a plot like this.



If we have oscilations (a perfect circle will result in a perfect circle.)



EX: consider the mass spring system.



$\ddot{x} + x = 0$

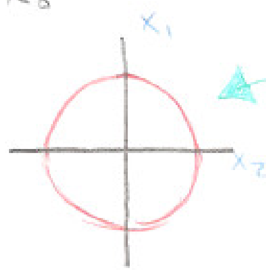
we can find state variables.

$x_1 = x \qquad x_1(t) = x_0 \cos(t)$

$x_2 = \dot{x} \qquad x_2(t) = -x_0 \sin(t)$

$x_1^2(t) + x_2^2(t) = x_0^2$

then is we plot...

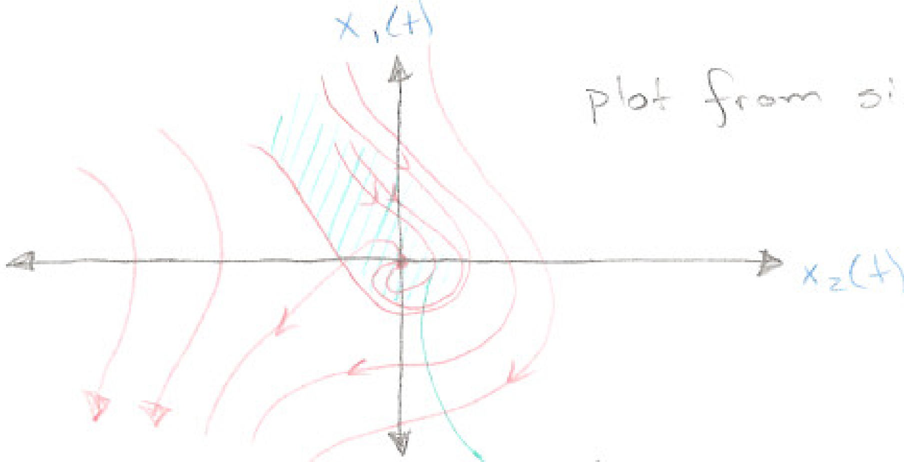


this is known as the phase portrait.

EX: $\ddot{x} + 0.6\dot{x} + 3x + x^2 = 0$

$x_1 = x \qquad \dot{x}_1 = x_2$

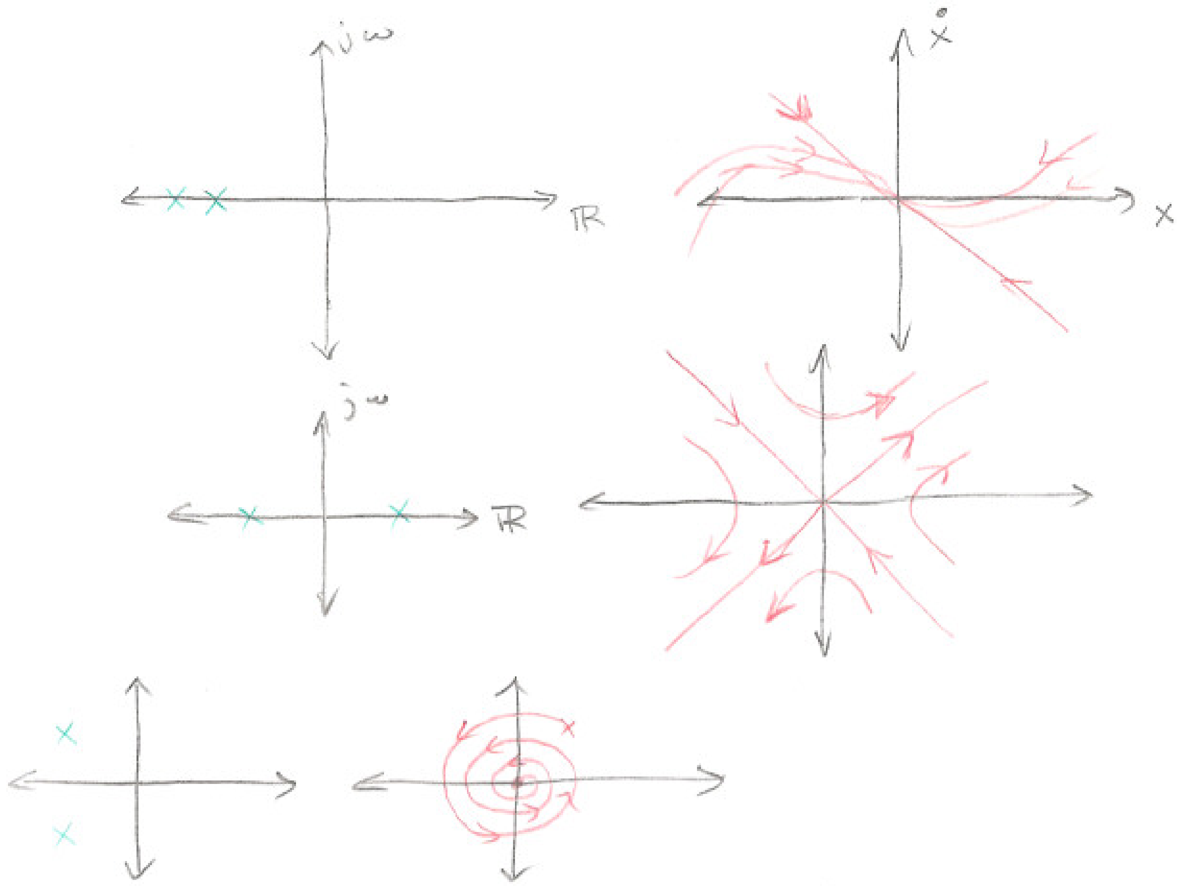
$x_2 = \dot{x} \qquad \dot{x}_2 = -0.6x_2 - x_1^2 - 3x_1$



plot from simulink

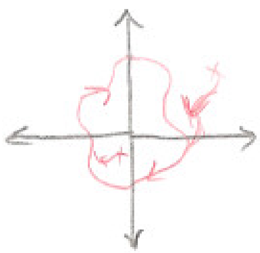
If you are in this region of attraction then the system converges to zero.

linear Systems

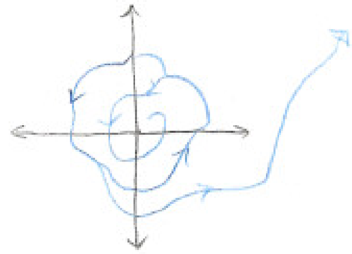


LIMIT CYCLES (phenomenian arising for non-linear systems)

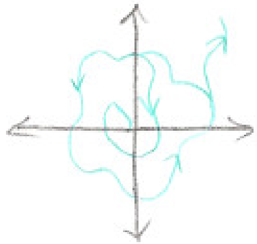
In the phase plane a limit cycle is defined as an isolated closed curve



A) stable limit cycle.



B) unstable limit cycle.



C) semi-stable limit cycle. (some converge others diverge)

Stable limit cycle: All trajectories in the limit cycle converge to it as t goes to infinity.

Unstable limit cycle: All trajectories in vacinity of the limit cycle diverge from it as t goes to infinity.

Semi Stable limit Cycle: Some trajectories converge while others diverge as $t \to \infty$